Fences and Functions Task

Claire decided to plant a rectangular garden in her back yard using 30 pieces of fencing that were given to her by a friend. Each piece of fencing was a vinyl panel 1 yard wide and 6 feet high. Claire wanted to determine the possible dimensions of her garden, assuming that she would use all of the fencing and did not cut any of the panels. She began by placing ten panels (10 yards) parallel to the back side of her house and then calculated that the other dimension of her garden then would be 5 yards, as shown in the diagram below.

1. Claire looked at the 10 fencing panels lying on the ground and decided that she wanted to consider other possibilities for the dimensions of the garden. In order to organize her thoughts, she let x be the garden dimension parallel to the back of her house, measured in yards, and let y be the other dimension, perpendicular to the back of the house, measured in yards. She recorded the first possibility for the dimensions of the garden as follows: When x = 10, y = 5.



- a. Make a table showing the possibilities for *x* and *y*.
- b. Find the perimeter of each of the possible gardens you listed in part a. What do you notice? Explain why this happens.
- c. Explain why y must be 5 when x is 10.
- d. Did you consider x = 15 in part a? If x = 15, what must y be? What would Claire do with the fencing if she chose x = 15?
- e. Can x be 16? What is the maximum possible value for x? Explain.
- f. Write a function relating the *y*-dimension of the garden to the *x*-dimension.

g. Make a graph of the possible dimensions of Claire's garden.

- h. What would it mean to connect the dots on your graph? Does connecting the dots make sense for this context? Explain.
- i. As the *x*-dimension of the garden increases by 1 yard, what happens to the *y*-dimension? Does it matter what *x*-value you start with? How do you see this in the graph? In the table? In your formula? What makes the dimensions change together in this way?

2. After listing the possible rectangular dimensions of the garden, Claire realized that she needed to pay attention to the area of the garden, because area determines how many plants can be grown.

- a. Does the area of the garden change as the *x*-dimension changes? Make a prediction, and explain your thinking.
- b. Use grid paper to make accurate sketches for at least three possible gardens. How is the area of each garden represented on the grid paper?



- c. Make a table listing all the possible *x*-dimensions for the garden and the corresponding areas. (To facilitate your calculations, you might want to include the *y*-dimensions in your table or add an area column to your previous table.)
- d. Make a graph showing the relationship between the *x*-dimension and the area of the garden. Should you connect the dots? Explain.

3. Because the area of Claire's garden depends upon the x-dimension, we can say that the area is a function of the x-dimension. Let's use G for the name of the function that uses each x-dimension an input value and gives the resulting garden area as the corresponding output value.

- a. Use function notation to write the function for the garden's area.
- b. What does G(11) mean? What is the value of G(11)?
- c. The set of all possible input values for a function is called the *domain* of the function. What is the domain of the garden area function *G*? In question 2d, how is your graph related to the domain?
- d. The set of all possible output values is called the *range* of the function. What is the range of the garden area function *G*? How can you see the range in your table? In your graph?
- e. As the *x*-dimension of the garden increases by 1 yard, what happens to the garden area? Does it matter what *x*-dimension you start with? How do you see this in the graph? In the table? Explain what you notice.
- f. What is the maximum value for the garden area, and what are the dimensions when the garden has this area? How do you see this in your table? In your graph?
- g. What is the minimum value for the garden area, and what are the dimensions when the garden has this area? How do you see this in your table? In your graph?
- h. In deciding how to lay out her garden, Claire made a table and graph similar to those you have made in this investigation. Her neighbor Javier noticed that her graph had symmetry. Your graph should also have symmetry. Describe this symmetry by indicating the line of symmetry. What about the context of the garden situation causes this symmetry?
- i. After making her table and graph, Claire made a decision, put up the fence, and planted her garden. If it had been your garden, what dimensions would you have used and why?